

8 October 2019

The sub orbit of the Virtual
Kähler-Ricci flow and a
cohomology theory for the
Riemann's Hypothesis

The way I formalize the proof of
Riemann's hypothesis is by representing
meromorphic function by a
solution to the virtual Kähler-
Ricci flow (more precisely the
trace of the generalized Laplacian)
Maybe this is a cohomology
theory, for to each meromorphic
function I try to associate
the sub orbit of a manifold.
More the ~~dimension~~ n^{th} cohomology
would correspond to the
dimension in which we work.

Remark: Note that I am trying to exactly represent a meromorphic function by a virtual Kähler-Ricci flow. Indeed I should approximate! The higher the dimension the better the approximation should be! Maybe there is a maximal dimension: above it the approximation does not get better. The way this would work is

$H_i \rightarrow H_{i-n}$
 $\uparrow \quad \quad \uparrow$
 thin in from dimension
 i to $i-n$ but fixed
 polynomial $j = P(g)$
 denoted by the Kähler-Ricci Flow

Bridgman note that (Extract from Wikipedia):
 "Let X be a smooth projective variety over a number field. The Bloch-Kato conjecture on values of L -functions predicts that the order of vanishing of an L -function of X at an integer point is equal to the rank of an abelian motive whose Galois group".
 So how this is related to my point of view for meromorphic functions! Actually maybe I should forget about meromorphic functions and concentrate on L -functions?

~~On zeros of the p-adic orbit~~ 13. 10. 2019
~~of the Ricci flow and the~~
~~approximation of meromorphic functions.~~
 Quizás los zeros de los orbitos
 es como del flujo ^{virtual} de Kähler-Ricci
 corresponden a valores de virtuales
 infinitamente curvas (de media).
 Estas serían especies singulares
 Hay de decir son bien g -o anteriormente
 hacen gran error. Querria representar

funciones meromorfas sobre la
curva C es C . Esto no representa
funciones meromorfas de
una sola variable! La curva
 C dependería por tanto de
de la posición sobre la variedad
virtual, no solamente del tiempo.
Una fibra por t se toma
la media correspondiente a cada
instante.

Ahora bien, volviendo al primer punto,
tendríamos un flujo ^{via π} de Kähler-Kric
y la órbita de π curva C a
media representaría una función
meromorfa. Ahora bien, cuando
la curva C media fuera cero
infinito, tendríamos un cero
de la función meromorfa.

Y cuando la curva C media

cero? Pero debería cada
la variedad C de media
infinitamente cercana. Esto
significa de C a un tipo muy
particular de singularidad de
la curva de Kähler-Kric.

Entendiendo cada una de las
variedades de C flujo π
dicho singularidad C no son
una descripción geométrica de
los años de dicha función.

Elijendo un flujo simple
representa la función π podría
también obtener una variedad
geométrica sobre la función.

Ahora bien, cuando dicha función no
pueda aproximarse por un
flujo, quizás una media boca
una media de aproximación.
Notar cómo esto resulta en una

cohomology.

29 Jan 2020

On the outer orbit of
the inter Kikk-kicci
flow

(A permanently point
of view)

En ^{with} ~~the~~ I consider
in Kikk-kicci flow
a lo major seria
ness internele co
~~voirdes~~ familia
pommilion de
voirdes alfabetic
reules y voir les
pommilion y ver

como otros muchos Kihlos.

En este caso habría por
introducir la noción de
curvatura de una variedad
afectiva (nótese por ejemplo
en símplos, i.e. no sería
una curvatura en el
sentido clásico).

Habría por ver si
funcionan también con
representables como
la traza de un
tensor de curvatura,
esto me influye
afectiva de Köhler.
Quiero añadir a los

cerca de este fin
viendo y hacer (nótese
que no incluye el
punto de vista de la
asociación).

10 Feb.

A statement on the
virtual-Kähler-Ricci
flow

Let $g(s) = P(g(s), Ric(s))$ (2)

take the case $\dim M = 1$,
i.e., one dimension.

$\nabla(s) = g(s)$ in which a
case for trace $(g(s)) = \nabla(s)$
in $\dim M = 1$.

Hence, it simplifies to

$$\nabla(s) = \sum_{i,j} c_{(i,j)} \nabla(s) Ric(s)^{i,j}$$

A good condition for
the well posedness
of this eq. is that,

$\nabla |s|$ in a bien
~~complex~~

(M, ∇) in a Kähler

$s=0$
~~is to~~
~~are~~

why? if $G=0$
 becomes it
 $s=0$
 "Flat"

$(M, \nabla(s^*))$
 in Kähler & Flat
 here s^* in

any vol of $\boxed{|\nabla |s||=0}$

Now ∇ ~~is~~ $\nabla = 1$

$$\Rightarrow \nabla |s| = \overline{\nabla |s|} = \nabla |s|$$

$\nabla |s|$ in a real
 function.

$$\Gamma_{00}^0 = \frac{1}{2} \left(\frac{\partial \nabla(s)}{\partial x} \right) \nabla(s)$$

$$\nabla(s) = f(x(s))$$

$$\Rightarrow \frac{d}{ds} \left(\frac{1}{2} \dot{x} \cdot \dot{x} \right) = \frac{1}{2} \frac{d}{ds} \left(\dot{x} \cdot \dot{x} \right)$$

Task: Use Spectral Sequence to
compute the Cohomology H^* .

Actually this is part of a
bigger picture: the jets.

Edford, Kopman, Johnson:
~~total~~ from jets to Discriminants.

Can we projectivize?
Can we make toric?

where we have done for convenience
 we can define the μ -function
 as

$$\mu(x, t) = \sum_{n \geq 0} \mu(x^{(n)}) t^n \in \mathbb{C}[[t]]$$

the n th symmetric
 product of x .

Does it make sense in $\mathbb{R}$?

Try to write my virtual Ricci flow
in the language of syntheticity.

Hodge Conjecture

$\Rightarrow$ Riemann Hypothesis?

Maybe use my Flows

Use motivic $\hookrightarrow$ Ricci
the usual det of measures instead of
~~the~~ $S(S)$

years in which I see close coll

All perspective correct /
variation without regular
point me Linder.

de Rham cohomology of a general complex manifold. On a compact Kähler manifold, the sequence degenerates, thereby leading to the Hodge decomposition of the de Rham cohomology.

Contents

Description of the spectral sequence

Degeneration

Purely algebraic proof

Non-commutative version

→ Study this problem
Apply this to the
virtual Floer

approximate

$$\sum N(M_t) s^t \stackrel{?}{=} \text{create using motivic types}$$

Plan:

ESCAVADO

$$No \quad g(s) \text{ now}$$

$\mu(M_t)$ in the normalized Euler characteristic it takes account each type ex. l. h. once and add it.

una familia de ~~complex~~ manifolds

Alternative: Motivic-~~zeta~~ like function

$$Z_p(M, s) = \sum_{n \geq 0} \sum_{g \in M_t} N(M_t) s^t$$

t parameter complejo.

$g \in$ es un manifold.

Then $K = K(H)$
 $\hookrightarrow$ Gauss curve $\rightarrow$ complex variety

$$\int_M K \text{ } dA + \int_{\partial M_t} K_g \text{ } ds = 2\pi i \chi(M_t)$$

bridge each topological shape
 $\rightarrow$ counted exactly once

$\Rightarrow$ we need a normalization: eg. 000000
 if we count 3 times
 $\Rightarrow \frac{3 \cdot 2}{3} = 2$

geo disc curve here
 the two functions (g_{t_1}, M_{t_1}) to (g_{t_2}, M_{t_2}) via diffeom.

this should be the zeta function of the category

Complete the $\mathbb{G} \times \mathbb{G} \times \mathbb{G}(H)$ of

this Euler characteristic where objects are (g, M_t) and $\bar{g} = P(g | Ric)$ the given is term of the Frobenius

The virtual-Kähler-Ricci flow

In order to prove, consider a ~~metric~~ $g = g(t)$ and g is constant
of time.

Also $n=1$ $\nabla(t) = g(z(t))$.

Suppose g is g -Ricci $\rightarrow$ depends on g

$$\dot{g} = \sum_{k,l} a_{k,l} g^k \text{Ric}^l(t)$$

$$\dot{g} = \frac{dg}{dt}$$

$$g' = \frac{\partial g}{\partial z}$$

Also, g -Ricci?

So the key is to use the Chernoff formula: $\Gamma_{z\bar{z}}^z = \Gamma =$

$$= \frac{1}{2} g^{-1} \cdot \left(\frac{\partial g}{\partial z} \right) = \frac{g'}{2g}$$

$$\Rightarrow \dot{g} = \sum_{k,l} a_{k,l} g^k \left(\frac{g'}{2g} \right)^l \Rightarrow$$

$$\dot{g} = \sum_{k,l} \frac{a_{k,l}}{2^l} g^{k-l} g'^l$$

$$\text{Also } \dot{g} = g' \cdot \dot{z} \Rightarrow$$

$$\dot{z} = \sum_{k,l} \frac{a_{k,l}}{2^l} g^{k-l} g'^{l-1}$$

Conclusión: Para los generadores de g (Ricci), tenemos $\text{Ric}^l = g^l$ es tal que g^l es g^l ...
No tiene sentido: g^l es g^l ...
No tiene sentido: g^l es g^l ...
No tiene sentido: g^l es g^l ...

The zeta function of a flow

Let M_t be a 1-parameter family of complex algebraic varieties.

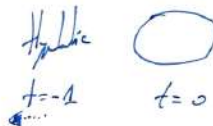
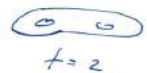
Given a fixed complex number, $\mu(M_{t=t_0}) :=$ Euler characteristic of $M_{t=t_0}$ or μ is a motivic measure

$$\zeta_M(s) := \sum \mu(M_t) s^t$$

$$\bar{\zeta}_M(s) = \sum \bar{\mu}(M_t) s^t$$

$\bar{\mu}$ is the normalized measure: if taken into account each topological type exactly once.

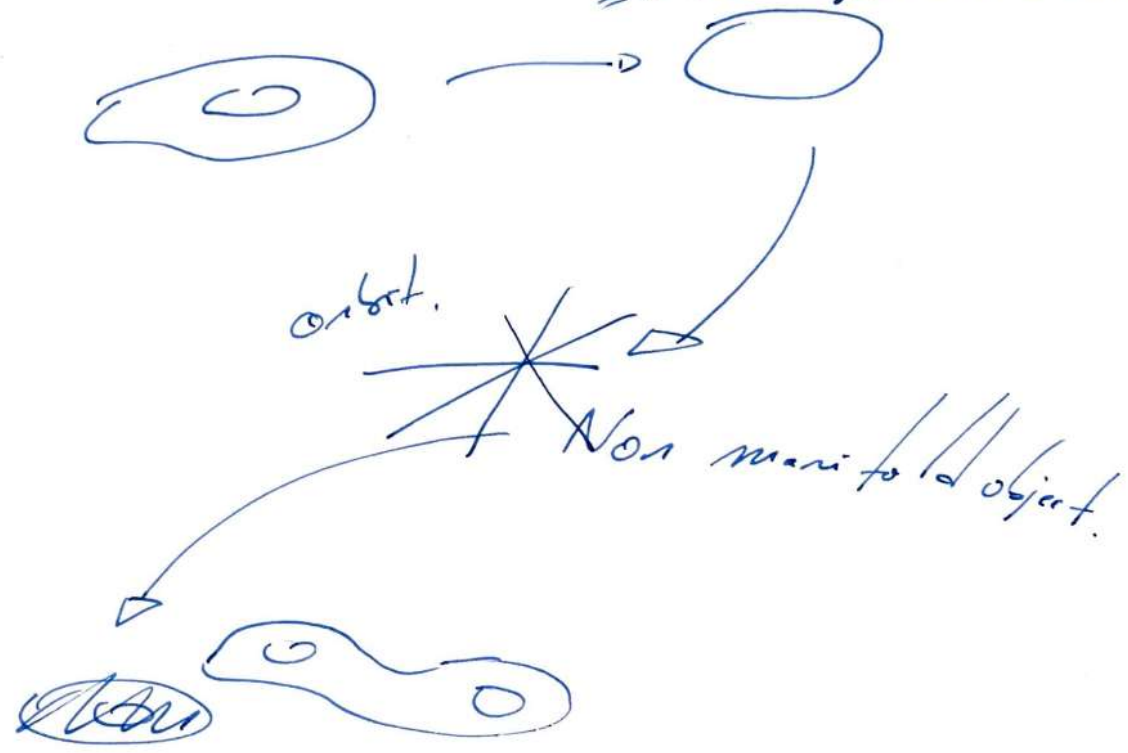
Example:



Instead of a virtual Kähler Ricci
flow, consider a sheaf of PDFs
→ Gauduchon - Kuran - Roch

Then sheaves cohomology

metric has to do with
the vector bundle (to put
spaces)
= a cohomology of sheaves.



so it is a stack : objects : generalised
complex varieties which a $\mathbb{C}^*$ -action
sing for metric.
groupoid = metrics?

If the metric is
Kähler-Einstein then
 $Ricci = \lambda g$
polynomially
rigid.

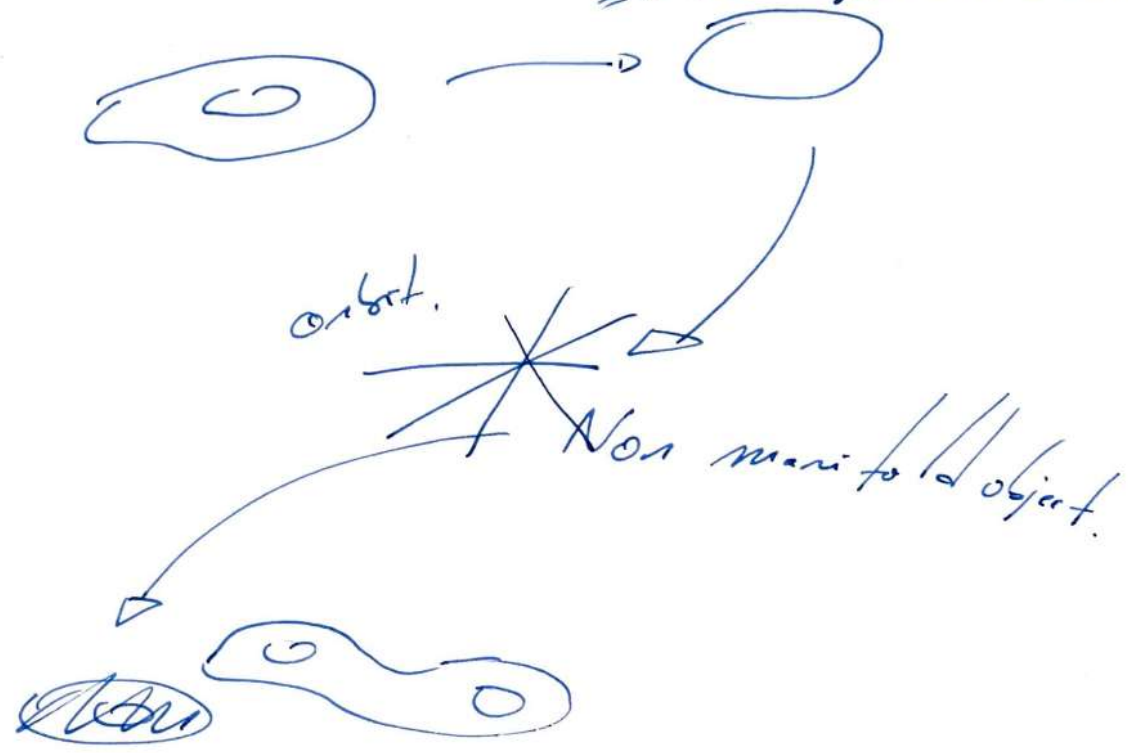
a Riemannian metric).

Look at the paper “CODIMENSION ONE FOLIATIONS ON HOMOGENEOUS VARIETIES” by VLADIMIRO BENEDETTI, DANIELE FAENZI, ALAN MUNIZ: Their distributions might be interesting for our study, for it gives an alternative way of dealing with the cases that arise in our setting, that is we might be able to avoid metrics.

Instead of a virtual Kähler Ricci
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The motive S_f is a flow

M_2 a k - R -parameters family of schemes of finite type over $\text{Spec } R$
for a (commutative/associative ring)

Sch_R category of schemes of finite type over $\text{Spec } R$

$\forall (R, \mu) \quad \mu: \text{Sch}_R \rightarrow \tilde{R}$ such that $\forall Y$ a closed subscheme of X
 $X \mapsto \mu(X)$
i) $\mu(X) = \mu(Y) + \mu(X-Y)$
ii) $\mu(X \times Y) = \mu(X) \mu(Y)$

Then $\text{Univ}(\mu): K_0(\text{Var}/R)$

That μ factors through $K_0(\text{Var}/R)$

Defn: X and X' have the same topological type $\Leftrightarrow \mu(X) = \mu(X')$

Defn: $\sum_{M_t}^*(s) = \sum_{\text{topol type}} \mu(M_t) s^t$ where s is a way of encoding a representative parameter t for each topological type.

Ex 1

M							
I	$\{t_1, t_2, t_3, t_4, t_5, t_6\}$	$\{t_1, t_2, t_3, t_4\}$	$\{t_1, t_2\}$	$\{t_1\}$	$\{t_1, t_2\}$	$\{t_1, t_2, t_3, t_4\}$	$\{t_1, t_2, t_3, t_4, t_5, t_6\}$
μ	$\mu = 6$	$\mu = 4$	$\mu = 2$	$\mu = 0$	$\mu = 2$	$\mu = 4$	$\mu = 6$
t^*	$t_1^* = -3$	$t_2^* = -2$	$t_3^* = -1$	$t_4^* = 0$	$t_5^* = 1$	$t_6^* = 2$	$t_7^* = 3$

$\sum_M^*(s) = \sum_{n \in \mathbb{Z}} 2n s^n$ alternative $\sum(s) = \sum \left(\frac{1}{i^n} \right) s = 2 \left(\frac{1}{2^n} \right) s$
 $= \sum_{n \in \mathbb{Z}} \frac{1}{n^n} + \sum_{n \in \mathbb{Z}} \frac{1}{n^n}$

Defn: We will also write, formally, an integral: $\sum_{M_t}(s) = \int X(M_t) d\mu := \sum \int \sum_{M_t}^*(s) dt$
 It is a subset of R with constant μ

Ex 1 (continued). $\sum_{M_t}(s) =$ It breaks up into sums with constant μ :
 $= \sum_n \int_{n \leq t \leq n+1} 2n s^n dt$ Express it as an object in $K_0(\text{Var}/R)$

a Riemannian metric).

Look at the paper “CODIMENSION ONE FOLIATIONS ON HOMOGENEOUS VARIETIES” by VLADIMIRO BENEDETTI, DANIELE FAENZI, ALAN MUNIZ: Their distributions might be interesting for our study, for it gives an alternative way of dealing with the cases that arise in our setting, that is we might be able to avoid metrics.

①

Dear Chris,
Dear Tim,
Dear Lampini,

In this letter I would like to explain a bit more in detail (but not more with rigour) what I mean by virtual Kähler-Ricci flow.

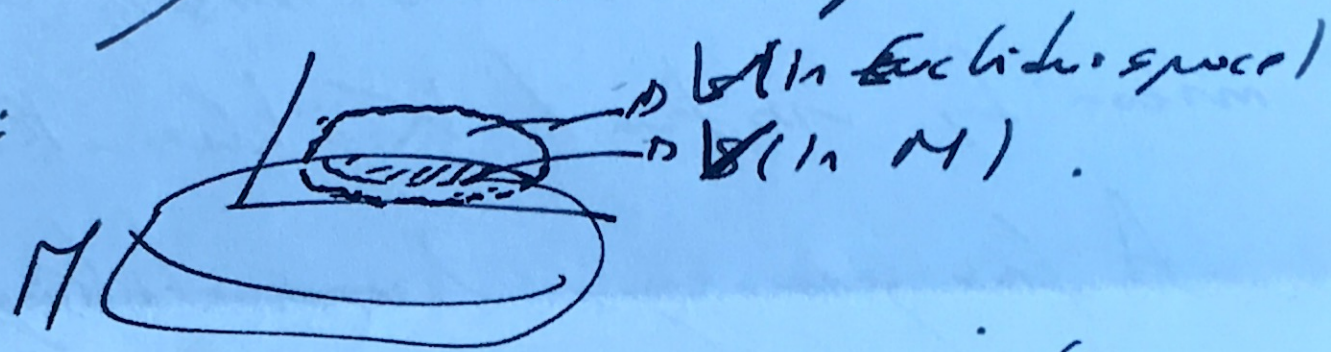
A few years ago I was taking the subject General Relativity. At the same time I was spending my free time thinking of a set of problems.

One of these problems was that I was not really understanding functions like the analytic continuation of ^(some) a Dirichlet series. So I had this naive idea: "why do not I see this function as being represented by a univariate family of manifolds?"

So, up to now, naively, a function $f(s)$ may or may not have a representation by a family of manifolds $M(s)$ ^{that} depending on a parameter s . For simplicity suppose that $f(s)$ is meromorphic. (The parameter s is complex.)

Now, each one of the elements $M(s^*)$ of $M(s)$ may or may not admit a metric $g(s^*)$. When $M(s)$ does admit a metric one can define some tensors. In particular, one can define the so called Ricci

curvature tensor. On a Riemannian manifold (that
 is, a real, smooth manifold, equipped with an inner
 product on the tangent space at each point that varies
 smoothly ^(the inner product) from point to point) the Ricci tensor
 measures the amount by which the volume
 of this manifold changes with respect to
 the Euclidean space:



The trace of the Ricci tensor is nothing
 more than the scalar curvature (or ∇)

So I was thinking that maybe I can
 represent $f(s)$ by $\nabla(s)$. Except that
 there will be values s^* of s for which
 $M(s)$ is not a Riemannian manifold and
 $\nabla(s)$ could not have a clear interpretation. As
 it happens, the Ricci tensor can still be defined
~~by a formula~~ for a special class of complex
 manifolds, called Kähler manifolds.
 However I do not expect $M(s^*)$ to be Kähler
 for every $s^* \in \mathbb{C}$. ~~After all~~ But this is just
 an intuition. If I am correct and $M(s^*)$ is
 not always Kähler, then I should
 allow a generalization of the Ricci tensor
 (or an extension there where it is not well defined)

(I can talk about Kähler
 in the next letter)

This is where the word "virtual" originates:
even if the Ricci tensor is not well defined
at s^* , I define the virtual Ricci tensor.

Anyway, years later, during my Ph.D. I
studied the Ricci flow which naturally gives
an interpretation for a family of manifolds
of a special class, and which was in fact
more similar to a proposal to solve the
Poincaré conjecture. In coordinates, the Ricci
flow is a tensorial PDE: $\frac{\partial}{\partial t} g_{ij} = -2 R_{ij}(S)$
metric Ricci tensor

I greatly ignore how Perelman used this
equation to prove the Poincaré conjecture.
But here is ~~the~~ ^a thing: ~~if~~ the Kähler
condition stays invariant under this equation.
Hence, if one starts with Kähler, one stays
with Kähler.

As I said, I do not expect $M(S^*)$ to be
always Kähler: I do not expect that any
monoplic function is the scalar curvature
of a family of Kähler manifolds. So why
don't we allow on the right hand side of
 $\frac{\partial}{\partial t} g_{ij}(s)$ to have any polynomial in

the variables g and R ? Let us call this polynomial P . Somehow the solutions of this problem should be vector complex functions in the variables. So let us suppose that the coefficients of P are meromorphic functions. I want a field On purpose I avoid writing $P \in M[g, R]$ because g and R are not independent variables: for each s^* there is a relation between them. Also it is not clear that this polynomial is well defined. At last, I do not want to define a multiplication: it may well be the Holomorphic product of tensors, or the matrix multiplication.

Now, given a meromorphic function $f(s)$, I would like to know whether there exist P such that $f(s) = \underbrace{\text{trace } R(s)}_{\text{vector and tensor}}$ and

$$g(s) = P(g, R).$$

If this is possible, can I relate s^* for which $M(s^*)$ is Kähler with the zeros of $g(s)$?

I attach the rigorous statement of the problem as well. I expect that next time in the next letter I can explore the Kähler condition. ~~But~~ Please ask me whatever you did not understand. I am writing.
Alex. 12 April 2019.

THE SCALAR ORBIT OF THE VIRTUAL KÄHLER-RICCI FLOW

ALEXANDRU IOSIF

ABSTRACT. We propose the virtual Kähler-Ricci flow as a framework for representing meromorphic functions.

Let (M, g_0) be a Kähler manifold of finite dimension n and let Ric_0 be its Ricci curvature tensor. Let P be a polynomial in the variables $g(s)$ and $\text{Ric}(s)$ and coefficients in the field $\mathbb{M}$ of meromorphic functions in the complex variable s . *→ some multiplication*

Definition 1. The *Virtual Kähler-Ricci Flow* $\mathfrak{R}_{P;(M, g_0)}$ of (M, g_0) is the set

$$\{(M, g(t)) : t \in \mathbb{C}, \frac{dg}{ds} = P(g(s), \text{Ric}(s)), g(0) = g_0\}.$$

As an interesting case, we reproduce the following result due to Bando.

Theorem 2. Every element of $\mathfrak{R}_{-2\text{Ric}(s);(M, g_0)}$ is a Kähler manifold.

Proof. If $P = -2\text{Ric}(s)$, then the virtual Kähler-Ricci Flow is a Kähler-Ricci flow. But the Kähler condition is preserved under the Kähler-Ricci flow. $\square$

Let $(M, g(t)) \in \mathfrak{R}_{P;(M, g_0)}$. If $g(t)$ is a positive definite Hermitian matrix at each point of M , then $\text{Ric}(t)$ is the Ricci curvature of $g(t)$. This motivates the following definition.

Definition 3. The *virtual Ricci curvature* is $\text{Ric}(t)$ and the *virtual scalar curvature* $\sigma(t)$ is $\text{tr}_{g(t)} \text{Ric}(t)$. The *Ricci orbit* of (M, g_0) is $\text{Ric}(s)$ and its *scalar orbit* is $\sigma(s)$.

The following problems are the motivation for the above definitions.

Problem 4. Given a meromorphic function $\mu(s)$, does there exist a Kähler manifold (M, g_0) and a P such that $\mu(s)$ is the scalar orbit of $\mathfrak{R}_{P;(M, g_0)}$ and for every zero t of $\mu(t)$ $g(t)$ is a real, positive definite, symmetric matrix? If so, then we can associate to each zero of $\mu(s)$ a flat Kähler manifold.

Problem 5. Given a Kähler manifold (M, g_0) , find a P , or prove that there is none, such that $(M, g(s))$ is Kähler if and only if $\sigma(s) = 0$.

Remark 6. It would be interesting to restrict this study to polynomials P such that the virtual Kähler-Ricci Flow has a unique solution. This would make the notion of orbit in the definition above a nicer object. A natural question that arises is the following: What is the algebraic structure of the set of polynomials P which lead to a virtual Kähler-Ricci Flow with a unique solution?

Date: December 14, 2017.

I have actually done some corrections after this date. However, the essential ideas were written at the end of 2017.

THE SCALAR ORBIT OF THE VIRTUAL KÄHLER-RICCI FLOW

ALEXANDRU IOSIF

ABSTRACT. We introduce the virtual Kähler-Ricci flow and propose it as a conjectural framework for representing meromorphic functions in one complex variable.

Let (M, g_0) be a Kähler manifold of finite dimension and let Ric_0 be its Ricci curvature tensor. Let P be a polynomial in the variables $g(s)$ and $\text{Ric}(s)$ and coefficients in the field $\mathbb{M}$ of meromorphic functions in one complex variable s . We would like to formally generalize the notion of Kähler-Ricci flow as follows.

Definition 1. The *virtual Kähler-Ricci flow* $\mathfrak{R}_{P;(M,g_0)}$ of (M, g_0) is

$$\{(M, g(t)) : t \in \mathbb{C}, \frac{dg}{ds} = P(g(s), \text{Ric}(s)), g(0) = g_0\}.$$

By setting $P = -2\text{Ric}(s)$, one obtains the following result due to Bando (e.g. [1, Section 1.1.3]).

Theorem 2 (Bando). *Every element of $\mathfrak{R}_{-2\text{Ric}(s);(M,g_0)}$ is a Kähler manifold.*

Proof. If $P = -2\text{Ric}(s)$, then the virtual Kähler-Ricci Flow is a Kähler-Ricci flow. But the Kähler condition is preserved under the Kähler-Ricci flow. $\square$

Let $(M, g(t)) \in \mathfrak{R}_{P;(M,g_0)}$. If $g(t)$ is a positive definite Hermitian matrix at each point of M , then $\text{Ric}(t)$ is the Ricci curvature of $g(t)$. This motivates the following generalization.

Definition 3. The *virtual Ricci curvature* is $\text{Ric}(t)$ and the *mean virtual scalar curvature* $\sigma(t)$ is the mean of $\text{tr}_{g(t)} \text{Ric}(t)$. The *Ricci orbit* of (M, g_0) is $\text{Ric}(s)$ and its *scalar orbit* is $\sigma(s)$.

The following problems are the motivation for the above generalizations.

Problem 4. *Given a meromorphic function $\mu(s)$, does there exist a Kähler manifold (M, g_0) and a polynomial P such that $\mu(s)$ is the scalar orbit of $\mathfrak{R}_{P;(M,g_0)}$ and, for every zero t of $\mu(t)$, $g(t)$ is a real, positive definite, symmetric matrix?*

If the answer to the previous problem were yes, then we could associate to each zero of $\mu(s)$ a (flat) Kähler manifold.

Problem 5. *Given a Kähler manifold (M, g_0) , find a P , or prove that there is none, such that $(M, g(s))$ is Kähler if and only if $\sigma(s) = 0$.*

It would be interesting to restrict this study to polynomials P such that the virtual Kähler-Ricci Flow has a unique solution. This would make the notion of orbit in the definition above a nicer object. A natural question that arises is the following.

Question 6. *What is the algebraic structure of the set of polynomials P which lead to a virtual Kähler-Ricci Flow with a unique solution?*

Note that a generalization of the Chern-Gauss-Bonnet theorem (or, more generally, of the Atiyah-Singer Theorem) is needed. It should be more general in the sense that it should cover the case of degenerated manifolds (that is, which fail to have a real curvature, or, in particular, a Hermitian metric).

Look at the paper “CODIMENSION ONE FOLIATIONS ON HOMOGENEOUS VARIETIES” by VLADIMIRO BENEDETTI, DANIELE FAENZI, ALAN MUNIZ: Their distributions might be interesting for our study, for it gives an alternative way of dealing with the cases that arise in our setting, that is we might be able to avoid metrics.

REFERENCES

1. Jian Song and Ben Weinkove, *An introduction to the Kähler-Ricci flow*, Springer, 2013, pp. 89–188.

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